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Level matrix, ^{16}N β decay, and the ^{12}C$(\alpha,\gamma)^{16}$O reaction

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The level matrix corresponding to the $\mathcal{K}$-matrix parametrization of a resonant nuclear reaction is derived and applied to the spectrum of α particles emitted following ^{16}N β decay. The parametrized spectrum is fitted to data simultaneously with the $E1$ capture cross section of the ^{12}C$(\alpha,\gamma)^{16}$O reaction and the p-wave phase shift of ^{12}C$(\alpha,\alpha)^{12}$C. Our analysis shows that new measurements of the α spectrum from ^{16}N β decay could be used to significantly reduce the uncertainty of the ^{12}C$(\alpha,\gamma)^{16}$O astrophysical S factor at 0.3 MeV. Various constraints on the parameters are analyzed and suggestions are made for further reducing the uncertainty in this crucial reaction rate.

I. INTRODUCTION

As a supplement to a recent paper [1] on the $\mathcal{K}$-matrix parametrization of resonant nuclear reactions, we derive here the level matrix associated with this parametrization. This, in turn, allows us to derive a parametrized form of the spectrum of α particles emitted after the β decay of unstable nuclei. When applied to the α particles emitted following ^{16}N β decay, the parametrization can be used to further constrain the astrophysical S factor for the ^{12}C$(\alpha,\gamma)^{16}$O capture reaction at 0.3 MeV, as compared to a recent analysis [2]. In the latter paper [2], only the cross section for the ^{12}C$(\alpha,\gamma)^{16}$O reaction and the elastic scattering p-wave phase shift for ^{12}C$(\alpha,\alpha)^{12}$C were parametrized in terms of a $\mathcal{K}$ matrix and fitted simultaneously to recent data. Unfortunately, fitting the phase shift proved to be only a weak constraint on the free parameters involved. In contrast, we show here that the simultaneous fit of the α spectrum from ^{16}N β decay is a very stringent constraint on the free parameters. This new $\mathcal{K}$-matrix analysis suggests that a new measurement of the energy spectrum of the α particles following ^{16}N β decay can significantly constrain the S factor for ^{12}C$(\alpha,\gamma)^{16}$O.

In Sec. II below we discuss the level matrix appropriate to the $\mathcal{K}$-matrix description. In Sec. III we give explicit formulas for the $\mathcal{K}$-matrix parametrization of the β-delayed α spectrum from ^{16}N. In Sec. IV we restate the corresponding formulas for the $E1$ ^{12}C$(\alpha,\gamma)^{16}$O cross section and ^{12}C$(\alpha,\alpha)^{12}$C p-wave phase shift, and fit all three types of data simultaneously. Finally, in Sec. V we discuss our results and present our conclusions. Throughout, our notation and definitions are those of Ref. [1].

II. THE LEVEL MATRIX

The particular form of the transition matrix $\mathcal{T}=1-\mathcal{S}$ corresponding to a one-level approximation of the $\mathcal{K}$ matrix has been given in Sec. VI of Ref. [1]. This is the simplest illustration of the fact that in the equation [1]

$$\mathcal{T}=-2ip\mathcal{K}(1-i\mu\mathcal{K})^{-1}p \tag{2.1}$$

the inversion of a channel matrix can be replaced by the inversion of a level matrix. We now obtain $\mathcal{T}$ in terms of a level matrix A (elements $A_{\lambda\mu}$) for any number of levels and channels.

The column vector g_λ has elements $g_{a\lambda}, g_{b\lambda}, \ldots$, and the diagonal matrix μ has diagonal elements $\mu_a, \mu_b, \ldots$, so that $h_\lambda=\mu g_\lambda$ is also a column vector with elements $\mu_a g_{a\lambda}, \mu_b g_{b\lambda}, \ldots$. With this notation we have [1]

$$\mathcal{K}=\sum_\lambda g_\lambda\times g_\lambda/(E_\lambda-E)\ , \tag{2.2}$$

$$1-i\mu\mathcal{K}=1-i\sum_\lambda h_\lambda\times g_\lambda/(E_\lambda-E)\ . \tag{2.3}$$

We now assume that the inverse of the latter quantity has the form

$$(1-i\mu\mathcal{K})^{-1}=1+i\sum_{\nu\mu} h_\nu\times g_\mu A_{\nu\mu}\ , \tag{2.4}$$

and justify it by obtaining the corresponding A matrix. The product of the matrices (2.3) and (2.4) being the unit matrix we must have

$$\sum_{\nu\mu} h_\nu\times g_\mu A_{\nu\mu}-\sum_\lambda h_\lambda\times g_\lambda/(E_\lambda-E)$$
$$-i\sum_{\lambda\nu\mu} A_{\nu\mu}(h_\lambda\times g_\lambda)(h_\nu\times g_\mu)/(E_\lambda-E)=0\ . \tag{2.5}$$

Since the matrix in the third term also reads

$$(h_\lambda\times g_\mu)m_{\lambda\nu}\ ,$$

with

$$m_{\lambda\nu}=\sum_e g_{e\lambda}h_{e\nu}=\sum_e \mu_e g_{e\lambda}g_{e\nu}\ , \tag{2.6}$$

Eq. (2.5) can be given the form

$$\sum_{\lambda\mu}(h_\lambda\times g_\mu)\left[A_{\lambda\mu}-\delta_{\lambda\mu}/(E_\lambda-E)\right.$$
$$\left.-i\sum_\nu m_{\lambda\nu}A_{\nu\mu}/(E_\lambda-E)\right]=0\ .$$

This equation is satisfied when

$$(E_\lambda - E)A_{\lambda\mu} - i\sum_\nu m_{\lambda\nu}A_{\nu\mu} = \delta_{\lambda\mu} , \tag{2.7}$$

or

$$\sum_\nu [(E_\lambda - E)\delta_{\lambda\nu} - im_{\lambda\nu}]A_{\nu\mu} = \delta_{\lambda\mu} , \tag{2.8}$$

i.e., when the symmetrical level matrix A, with elements $A_{\lambda\mu}$, is defined by its inverse,

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - i\sum_{e^+} p_e^2 g_{e\lambda} g_{e\mu} , \tag{2.9}$$

since in $m_{\lambda\mu}$, according to Sec. VI of Ref. [1], we have $\mu_{e^+} = p_e^2$, $\mu_{e^-} = 0$ in open and closed channels, respectively.

From Eqs. (2.2), (2.4), and (2.7) we obtain

$$\begin{aligned}\mathcal{K}(1 - i\mu\mathcal{K})^{-1} &= \sum_\lambda g_\lambda \times g_\lambda/(E_\lambda - E) \\ &\quad + i\sum_{\lambda\mu\nu} g_\lambda \times g_\mu m_{\lambda\nu}A_{\nu\mu}/(E_\lambda - E) \\ &= \sum_{\lambda\mu} g_\lambda \times g_\mu A_{\lambda\mu}\end{aligned} \tag{2.10}$$

and

$$\mathcal{T} = -2i\sum_{\lambda\mu} p(g_\lambda \times g_\mu)pA_{\lambda\mu} . \tag{2.11}$$

For an integrated cross section, and with partial widths defined as in Ref. [1] by $\Gamma_{c\lambda} = 2p_c^2 g_{c\lambda}^2$, we have

$$\sigma_{cd}^J = \frac{4\pi g^J}{k_c^2}\left|\sum_{\lambda\mu} p_c g_{c\lambda} A_{\lambda\mu} g_{d\mu} p_d\right|^2 \tag{2.12}$$

$$= \frac{\pi g^J}{k_c^2}\left|\sum_{\lambda\mu} \Gamma_{c\lambda}^{1/2} A_{\lambda\mu} \Gamma_{d\mu}^{1/2}\right|^2 . \tag{2.13}$$

The result (2.10) is only formally the same as in R-matrix theory [3], the main difference being in the very definition (2.9) of the level matrix A. Here, $i\mu_e$, which corresponds to $L_e^0 = S_e + iP_e - B_e$ in R-matrix theory, is not only independent of the channel radii, but it also vanishes in all closed channels e^- with two charged fragments, as seen in Sec. VI of Ref. [1].

Nevertheless, from the very form of Eqs. (2.10)–(2.12), we can infer that the applications, which have made use of the level matrix associated with the R matrix, should also be feasible with the level matrix associated with the $\mathcal{K}$ matrix. This should hold in particular for the analysis of the energy spectrum of the α particles following the β^- decay of ^{16}N, whose 2^- ground state is unstable. This and related problems have been analyzed previously by Barker *et al.* [4–6] in papers based on an R-matrix parametrization.

III. THE SPECTRUM OF α PARTICLES FOLLOWING ^{16}N β DECAY

For the ^{12}C(α,γ)^{16}O reaction, the astrophysical factor

$$S(E) = E\sigma_{\alpha\gamma}(E)\exp(2\pi\eta) \tag{3.1}$$

is of particular interest at the "most effective" ^{12}C+α center-of-mass energy $E = 0.3$ MeV. However, a simultaneous fit to the capture cross section $[\sigma_{\alpha\gamma}(E)]$ and the elastic-scattering ^{12}C(α,α)^{12}C data fixes $S(0.3)$ only within a wide range [2]. It is therefore appropriate to perform a new $\mathcal{K}$-matrix analysis with the further constraint of simultaneously fitting the spectrum of α particles following ^{16}N β decay.

The required fit can be performed by applying Eqs. (2.10)–(2.13) for the $E1$ capture and using a three-level approximation. The first level is at $E_1 = -0.0451$ MeV (the energy of the 1^- bound state of ^{16}O), the second level at E_2 corresponds to the broad 1^- resonance at $E \simeq 2.45$ MeV ($E_x \simeq 9.61$ MeV), while E_3 is associated with background contributions accounting for levels at higher excitation.

The ground state of ^{16}N is 2^-, while ^{12}C and the α particle are both 0^+. Accordingly, if only allowed Gamow-Teller transitions are considered, the corresponding α particles are emitted by ^{16}O* in 1^- and 3^- states. According to Barker and Warburton [6], the parametrized form for the α particle spectrum in the 1^- ^{12}C+α channel has a form corresponding to Eq. (2.12) in which appropriate feeding factors for each energy level are substituted for the factors related to the entrance channel c. With the index α being used to characterize the 1^- ^{12}C+α exit channel, this gives for the number of α particles per unit energy interval and with $l = 1$ angular momentum

$$N_{1\alpha}(E) = f_\beta(E)\left|\sum_{\lambda\mu} B_\lambda A_{\lambda\mu} g_{\alpha\mu} p_{1\alpha}\right|^2 \tag{3.2a}$$

$$= \tfrac{1}{2} f_\beta(E)\left|\sum_{\lambda\mu} B_\lambda A_{\lambda\mu} \Gamma_{\alpha\mu}^{1/2}\right|^2 , \tag{3.2b}$$

where

$$f_\beta(E) = f(W_0, 8) \tag{3.3}$$

is the integrated Fermi function [7] with $Z = 8$ and $W_0 = (3.768 - E)/m_e$, $E \le E_{\max} = 3.257$ MeV, while the B_λ are feeding factors proportional to the Gamow-Teller matrix elements between the initial and final hadronic states.

In Eqs. (3.2) the elements of the level matrix A are implicitly defined by Eq. (2.9) and here we have

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - i(p_{1\alpha}^2 g_{\alpha\lambda} g_{\alpha\mu} + p_{1\gamma}^2 g_{\gamma\lambda} g_{\gamma\mu}) . \tag{3.4}$$

We drop the index $l = 1$ when no confusion can arise. In Eq. (3.4) we can neglect the contribution from the γ channel to the last term and write

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - ip_{1\alpha}^2 g_{\alpha\lambda} g_{\alpha\mu} \tag{3.5}$$

since, from Table I in Ref. [2], the neglected terms are about 6 orders of magnitude smaller than those associated with the α channel.

The inversion of A^{-1} is easily performed. Defining

$$D = (E_1 - E)(E_2 - E)(E_3 - E)(1 - ip_{1\alpha}^2 \mathcal{K}_{1\alpha\alpha}) , \tag{3.6}$$

with

$$\mathcal{K}_{1\alpha\alpha} = \sum_{\lambda=1}^{3} g_{\alpha\lambda}^2/(E_\lambda - E) , \tag{3.7}$$

we obtain

$$A_{\lambda\mu}=\frac{\delta_{\lambda\mu}}{E_\lambda-E}\frac{1}{1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha}}+\mathcal{A}_{\lambda\mu}\ , \tag{3.8}$$

with

$$D\mathcal{A}_{11}=-ip_{1\alpha}^2[g_{\alpha2}^2(E_3-E)+g_{\alpha3}^2(E_2-E)]\ ,$$
$$D\mathcal{A}_{12}=-ip_{1\alpha}^2 g_{\alpha1}g_{\alpha2}(E_3-E)\ , \tag{3.9}$$

and all the other $\mathcal{A}_{\lambda\mu}$ being obtained by circular permutation. Hence

$$\sum_\mu \mathcal{A}_{\lambda\mu}g_{\alpha\mu}=0 \tag{3.10}$$

and, instead of Eq. (3.2a), with the approximation (3.5), we simply have

$$N_{1\alpha}(E)=f_\beta(E)p_{1\alpha}^2(E)\left|\frac{B_1g_{\alpha1}/(E_1-E)+B_2g_{\alpha2}/(E_2-E)+B_3g_{\alpha3}/(E_3-E)}{1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha}}\right|^2\ , \tag{3.11}$$

a formula similar to the R-matrix expression used by Barker in his early paper [5].

In Eq. (3.11), B_2 and B_3 are free parameters obtained from fitting the energy spectrum of emitted α particles. However, B_1 can be obtained from the β-delayed γ-ray intensity from the E_1 bound state, since when the total lepton energy is larger than 3.257 MeV, E is negative, the α channel is closed for the decay of $^{16}O^*(1^-)$, and only the γ channel is open. With $Q=7.1616$ MeV and

$$p_{1\gamma}^2=[(Q+E)/\hbar c]^3,\quad \Gamma_{\gamma\mu}=2p_{1\gamma}^2g_{\gamma\mu}^2\ , \tag{3.12a}$$

$$(A^{-1})_{\lambda\mu}=(E_\lambda-E)\delta_{\lambda\mu}-ip_{1\gamma}^2g_{\gamma\lambda}g_{\gamma\mu}\ , \tag{3.12b}$$

the γ spectrum is given by

$$N_{1\gamma}(E)=\tfrac{1}{2}f_\beta(E)\left|\sum_{\lambda\mu}B_\lambda A_{\lambda\mu}\Gamma_{\gamma\mu}^{1/2}\right|^2\ . \tag{3.13}$$

Since the γ widths are very small, so is $N_{1\gamma}(E)$, except when $E\simeq E_1$. Hence, we are justified in using a one-level approximation to Eq. (3.13). With $A_{11}=1/(E_1-E-ip_{1\gamma}^2g_{\gamma1}^2)$ we have

$$N_{1\gamma}(E)=\tfrac{1}{2}f_\beta(E)B_1^2\frac{\Gamma_{\gamma1}}{(E-E_1)^2+\Gamma_{\gamma1}^2/4}\ . \tag{3.14}$$

To a very good approximation, because $\Gamma_{\gamma1}(E_1)$ is only 55 meV, this gives for the total number of γ's emitted

$$N_{1\gamma}=\int_{-Q}^{0}N_{1\gamma}(E)dE=\pi B_1^2f_{\beta1}\ , \tag{3.15}$$

with $f_{\beta1}=f_\beta(E_1)=f(7.462,\,8)$, and hence

$$B_1^2=\frac{N_{1\gamma}}{\pi f_{\beta1}}\ . \tag{3.16}$$

Because E_2 is a broad resonance, a similar evaluation does not apply to B_2. We note, however, that $N_{1\alpha}/\pi f(2.581,\,8)$ obtained from the data has the correct order of magnitude and hence is a good starting value for B_2 in the search for the best fit.

IV. FITTING σ_{E1}, δ_1, AND $N_{1\alpha}$ TO THE DATA

The parametrized expressions for the $E1$ capture cross section and the δ_1 phase shift to be fitted to data are the same as those in Ref. [2]. With [8]

$$\mathcal{K}_{1\alpha\alpha}=\frac{g_{\alpha1}^2}{E_1-E}+\frac{g_{\alpha2}^2}{E_2-E}+\frac{g_{\alpha3}^2}{E_3-E}+b_{\alpha\alpha}\ , \tag{4.1}$$

$$\mathcal{K}_{1\gamma\alpha}=\frac{g_{\gamma1}g_{\alpha1}}{E_1-E}+\frac{g_{\gamma2}g_{\alpha2}}{E_2-E}+\frac{g_{\gamma3}g_{\alpha3}}{E_3-E}+b_{\gamma\alpha}\ , \tag{4.2}$$

they read

$$\sigma_{E1}=\frac{12\pi}{k^2}p_{1\alpha}^2p_{1\gamma}^2|\mathcal{K}_{1\gamma\alpha}/(1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha})|^2\ , \tag{4.3}$$

$$\delta_1=\arctan(p_{1\alpha}^2\mathcal{K}_{1\alpha\alpha})\ . \tag{4.4}$$

Here, as in Ref. [2], a single background pole term in $\mathcal{K}_{1\alpha\alpha}$ does not lead to a sufficiently good fit of δ_1. Rather than introducing as in Ref. [9] a second background pole term with a very large pole energy, we simply add a constant term $b_{\alpha\alpha}$ as we did in Ref. [2]. Similarly a constant $b_{\gamma\alpha}$ is added to $\mathcal{K}_{1\gamma\alpha}$. These background terms are introduced as a low-energy approximation of the contributions from distant levels E_λ ($\lambda>2$). In order to constrain as much as possible the background parameters, we have, as in Ref. [2], used the presently available data over the widest possible energy range, namely, up to $E=4.9$ MeV for the phase shift δ_1, $E=2.9$ MeV for the capture cross section σ_{E1}, and $E=2.7$ MeV ($E_\alpha=3.6$) for the α spectrum.

As in Ref. [2], δ_1 is fitted simultaneously to three sets of data, [9] while here, for illustration, σ_{E1} is fitted only to the Kremer *et al.* [10] data. For the experimental α spectrum, we use as in Ref. [12], the data of Hättig *et al.* [11] as obtained by Barker [4,13], with counts corresponding to the $l=3$ α channel subtracted from the total α spectrum. This reduces the total number of counts from $N_\alpha=3.24\times10^7$ to $N_{1\alpha}=3.15\times10^7$, where the latter number is to be used in the evaluation of the feeding factor B_1 as given by Eq. (3.16). This is accomplished using

$$N_{1\gamma}=N_{1\alpha}Y_{1\gamma}(E_1)/Y_{1\alpha}(E_2)\ , \tag{4.5}$$

where the branching ratios are [14] $Y_{1\gamma}(E_1)=0.048\pm0.004$, $Y_{1\alpha}(E_2)=(1.20\pm0.05)\times10^{-5}$, and thus we obtain

TABLE I. Parameter values for the best fits with $g_{\gamma 3}, b_{\gamma\alpha}$ as free parameters (second column) and with $g_{\gamma 3}=b_{\gamma\alpha}=0$ (third column). The numbers in parentheses are fixed parameters and have been obtained from earlier work (see Ref. [2]) and Eq. (4.6). To give the reduced with amplitudes their usual dimensions, they have been multiplied by $a^{-3/2}$, with $a=5.46$ fm as in Ref. [2].

E_1(MeV)	(-0.0451)	(-0.0451)
$g_{\alpha 1}a^{-3/2}$(MeV$^{1/2}$)	-5.34	-5.89
$g_{\gamma 1}a^{-3/2}$(MeV$^{1/2}$)	(1.897×10^{-3})	(1.897×10^{-3})
B_1	(6804)	(6804)
E_2(MeV)	2.452	2.453
$g_{\alpha 2}a^{-3/2}$(MeV$^{1/2}$)	7.02	6.97
$g_{\gamma 2}a^{-3/2}$(MeV$^{1/2}$)	0.659×10^{-3}	0.632×10^{-3}
B_2	-2385	-2365
E_3(MeV)	(7.000)	(7.000)
$g_{\alpha 3}a^{-3/2}$(MeV$^{1/2}$)	$12.00i$	$12.04i$
$g_{\gamma 3}a^{-3/2}$(MeV$^{1/2}$)	$-2.66\times10^{-3}i$	—
B_3	$4028i$	5116
$b_{\alpha\alpha}a^{-3}$	70.83	72.12
$b_{\gamma\alpha}a^{-3}$	-5.71×10^{-3}	—
$\Gamma_{\gamma 1}$(MeV)	(55×10^{-9})	(55×10^{-9})
$\Gamma_{\alpha 2}$(MeV)	0.467	0.461
$\Gamma_{\gamma 2}$(MeV)	16.4×10^{-9}	15.0×10^{-9}
$S_{E1}(0.3)$(MeV b) at $\chi^2_{\rm min}$	0.043	0.055
$S_{E1}(0.3)$(MeV b) range	0.027–0.063	0.038–0.074

$$B_1=6804\pm635\ . \tag{4.6}$$

Other fixed parameters are E_1, E_3, $g_{\gamma 1}$, as in Ref. [2].

We fitted simultaneously σ_{E1}, δ_1, and $N_{1\alpha}$ as given by Eqs. (4.3), (4.4), and (3.11) to the three sets of data. We define an effective χ^2 by [2]

$$\chi^2_{\rm eff}=\tfrac{1}{3}(\chi^2_\gamma+\chi^2_\delta+\chi^2_\beta)\ , \tag{4.7}$$

where $\chi^2_\gamma,\chi^2_\delta,\chi^2_\beta$ are χ^2 per data point for the $E1$ capture cross section, the $l=1$ phase shift and the $l=1$ α spectrum, respectively. Our best fit corresponds to $\chi^2_{\rm min}$, the minimum of $\chi^2_{\rm eff}$. The numerical results for the best fit are given in Table I, while Fig. 1 gives $\chi^2_{\rm eff}$ versus $S_{E1}(0.3)$ when this quantity is used as a free parameter instead of $g_{\alpha 1}$, as discussed in Ref. [2]. The $\chi^2_{\rm eff}$ is minimized at $S_{E1}(0.3)=0.043$ MeV b. From Fig. 1 we also see that the range of acceptable values for $S_{E1}(0.3)$, defined as those whose $\chi^2_{\rm eff}$ does not exceed $\chi^2_{\rm min}$ by more than 30% is 0.027–0.063 MeV b. Ten free parameters and 106 data points are involved in this fit.

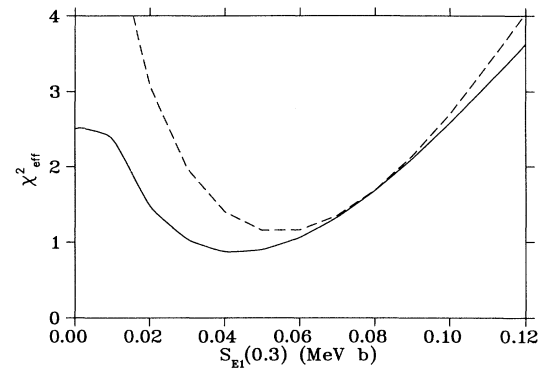

FIG. 1. The effective χ^2 vs $S_{E1}(0.3)$. The two curves correspond to fits without constraint on the free parameters (solid line), and with $g_{\gamma 3}=b_{\gamma\alpha}=0$ (dashed line).

Fits with fewer free parameters have also been obtained by introducing constraints corresponding to those used by Barker [5,15] in his R-matrix fits. They are $g_{\gamma 3}=b_{\gamma\alpha}=0$, and $B_3=0$. Complete results with $g_{\gamma 3}=b_{\gamma\alpha}=0$ are also given in Table I, while for $B_3=0$, only the main results are reported in Table II, together with those obtained in Ref. [2]. As seen from Fig. 1, the constraints $g_{\gamma 3}=b_{\gamma\alpha}=0$ shift the range of acceptable $S_{E1}(0.3)$ values slightly higher, while the allowed range (0.038–0.074 MeV b) is not reduced. These constraints on the γ channel parameters mainly increase χ^2_γ, from 0.88 to 1.48. This suggests that the parametrization of σ_{E1} is then less reliable, and hence, so should be the astrophysical factor $S_{E1}(0.3)$.

The constraint $B_3=0$ increases χ^2_β by more than a factor of 10 as seen in Table II, an unacceptably large increase. With both constraints, $\chi^2_\gamma=4.92$ is also unacceptable.

V. DISCUSSION OF THE RESULTS AND CONCLUSIONS

The overall situation could be much improved by better data for σ_{E1} in the 1–3-MeV energy range along with data points below and above this range, but this does not appear likely in the near future. However, a new measurement of the α spectrum extending below and above the energy range of the Hättig *et al.* [11] data, $E=1.5$–2.7 MeV, appears more feasible and has recent-

TABLE II. Values of χ^2 for simultaneous fits to the three sets of data, with different constraints, and the corresponding range of allowed values for $S_{E1}(0.3)$. For the sake of comparison, also given in the first line are the results obtained in Ref. [2], where the α spectrum from ^{16}N β decay was not fitted.

Constraints	χ^2_γ	χ^2_δ	χ^2_β	$\chi^2_{\rm min}$	$S_{E1}(0.3)$ range
None (from Ref. [2])	0.92	1.34	—	1.13	0.00–0.16
None	0.88	1.51	0.20	0.86	0.027–0.063
$g_{\gamma 3}=b_{\gamma\alpha}=0$	1.48	1.57	0.35	1.13	0.038–0.074
$B_3=0$	0.99	1.91	2.17	1.69	
$g_{\gamma 3}=b_{\gamma\alpha}=B_3=0$	4.92	2.25	1.99	3.05	

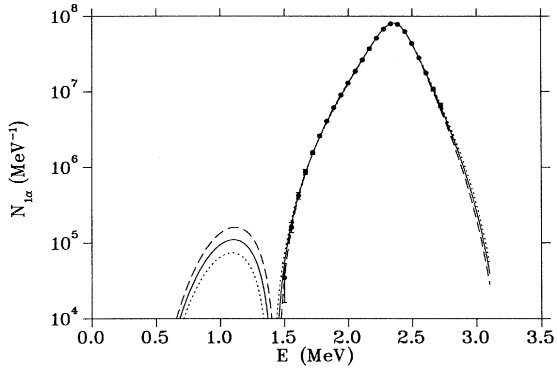

FIG. 2. The parametrized α spectrum vs center-of-mass energy. The solid line gives the α spectrum when σ_{E1}, δ_1, and $N_{1\alpha}$ are fitted simultaneously without constraint. The others give the α spectrum when $S_{E1}(0.3)$ is given the extreme values of its allowed range, i.e., $S_{E1}(0.3)=0.027$ MeV b for the dotted line, and $S_{E1}(0.3)=0.063$ MeV b for the dashed line. All three spectra vanish at $E \approx 1.4$ MeV.

ly been proposed at TRIUMF [16]. In this context, the parametrized spectra in Fig. 2 are shown well above and below the range of the present data. One spectrum corresponds to the best fit with $S_{E1}=0.043$ MeV b, while the others correspond to $S_{E1}(0.3)=0.027$ and 0.063 MeV b, respectively; i.e., the end points of the range of acceptable values of $S_{E1}(0.3)$ deduced from Fig. 1. The computed spectra have two maxima because the numerator in Eq. (3.11) vanishes at $E \simeq 1.4$ MeV. The main parameters involved in these fits are reported in Table III. The parameters $g_{\alpha 2}$ and B_2 are nearly the same for the three fits. In contrast, from the fit with $S_{E1}(0.3)=0.027$ MeV b to that with $S_{E1}(0.3)=0.063$ MeV b, $g^2_{\alpha 1}$ increases by as much as a factor of 2. But this large variation is partially compensated by an important increase in $|B_3|$. Nevertheless, with $S_{E1}(0.3)=0.063$ MeV b, the number of counts at the energy of the first maximum ($\simeq 1.1$ MeV) increases by 45% relative to that when $S_{E1}(0.3)=0.043$ MeV b. With $S_{E1}(0.3)=0.027$ MeV b, it is lowered by 33%. Even a 10% error in the number of counts at energies near 1.1 MeV could result in an important reduction in the range of acceptable values for $g_{\alpha 1}$ and hence $S_{E1}(0.3)$. This analysis of our results at low energy agrees with Baye and Descouvemont [17]. They emphasized the strong correlation between the R-matrix reduced width $\gamma^2_{\alpha 1}$ of the E_1 bound state and the α spectrum in the 0.8–1.2-MeV energy range.

In all of our fits we have kept B_1 fixed, although we have estimated that an uncertainty close to 10% must be attached to the numerical value $B_1=6804$. Since B_1 and $g_{\alpha 1}$ are strongly correlated in the energy spectrum (3.11), it is also desirable to improve the precision of the numerical value of B_1. According to Eqs. (3.16) and (4.5), this requires a better determination of the branching ratios $Y_{1\gamma}(E_1)$ and $Y_{1\alpha}(E_2)$, if no direct measurement is made of the β-delayed γ-ray intensity from the E_1 bound state.

Turning to the high-energy part of the α spectrum, new data extending to energies higher than $E=2.7$ MeV should better constrain the B_3 feeding factor. As seen earlier, this in turn could better constrain $g_{\alpha 1}$, since the term in B_3 is not negligible at 1.1 MeV. At $E=3$ MeV, with $S_{E1}(0.3)=0.063$ MeV b, the parametrized spectrum is reduced by 22% relative to that with $S_{E1}(0.3)=0.043$ MeV b. With $S_{E1}=0.027$ MeV b, it increases by 24%. These variations are significant, but as seen in Fig. 2, at $E=3$ MeV, the spectrum varies rapidly with E. Hence, extending the range of the data up to 3 MeV might not be as useful in constraining the parametrization as data taken near 1 MeV.

When our results are compared with those of Barker, [4,15,18] it appears from Table II that the constraints we have applied ($g_{\gamma 3}=b_{\gamma\alpha}=0$ and $B_3=0$) are more stringent than his. This may be related to the fact that when, e.g., we introduce the $B_3=0$ constraint in the α spectrum, the corresponding term disappears completely from the parametrization. This is not the case in Barker's parametrization. An R-matrix many-level fit is complicated by the fact that any physical constraint associated with a level (energy, reduced width amplitude, or feeding factor) must be applied only when the boundary condition constant is chosen to have a vanishing energy shift at that same level. In this context, it is obvious that the absence of boundary condition constants in the $\mathcal{K}$-matrix parametrization greatly simplifies the fitting procedure. A single fit contains all the physical parameters, resonance energies, reduced width amplitudes, and feeding factors.

We can conclude that the parametrized spectrum (3.11) of the α particles from ^{16}N β decay, with its three feeding factors, allows a very good fit to the present data. This

TABLE III. Variation of several key parameters for the fits with $S_{E1}(0.3)$ within its allowed range. Note the strong correlation between $S_{E1}(0.3)$ and $g_{\alpha 1}$, B_3, and the α spectrum at 1.1 and 3.0 MeV.

$S_{E1}(0.3)$ (MeV b)	0.063	0.043	0.027
$g_{\alpha 1}a^{-3/2}$(MeV$^{1/2}$)	-6.33	-5.34	-4.43
$g_{\alpha 2}a^{-3/2}$(MeV$^{1/2}$)	6.96	7.02	7.09
B_2	-2370	-2385	-2399
$g_{\alpha 3}a^{-3/2}$(MeV$^{1/2}$)	$12.39i$	$12.00i$	$11.69i$
B_3	$5724i$	$4028i$	$2324i$
$N_{1\alpha}$ at 1.1 MeV	+45%	Reference	-33%
$N_{1\alpha}$ at 3 MeV	-22%	Reference	+24%

introduces a very stringent constraint on the parameters of the $\alpha+{}^{12}$C channel involved in the $E1$ capture cross section. The value we have obtained for the $E1$ part of the astrophysical factor is

$$S_{E1}(0.3)=0.043^{+0.020}_{-0.016}\ \mathrm{MeV\,b}\ . \tag{5.1}$$

If we take for the $E2$ part of the S factor the result obtained in Ref. [2],

$$S_{E2}(0.3)=0.007^{+0.024}_{-0.005}\ \mathrm{MeV\,b}\ , \tag{5.2}$$

we obtain for the total S factor

$$S(0.3)=0.05^{+0.03}_{-0.02}\ \mathrm{MeV\ b}\ . \tag{5.3}$$

However, the degree of confidence one can have in the results (5.1) and (5.3) is limited by the fact that we have not included in our fit the f-wave part of the α spectrum. In addition, since the original experiment [11] did not attempt to accurately extract that α spectrum, there are potential uncertainties associated with the detector response and resolution [19].

We have given arguments justifying the importance of remeasuring the α spectrum of ^{16}N β decay over a wider energy range, and also of obtaining better branching ratios for the β decay to the 7.12-MeV bound state and the 9.61-MeV resonance of ^{16}O.

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